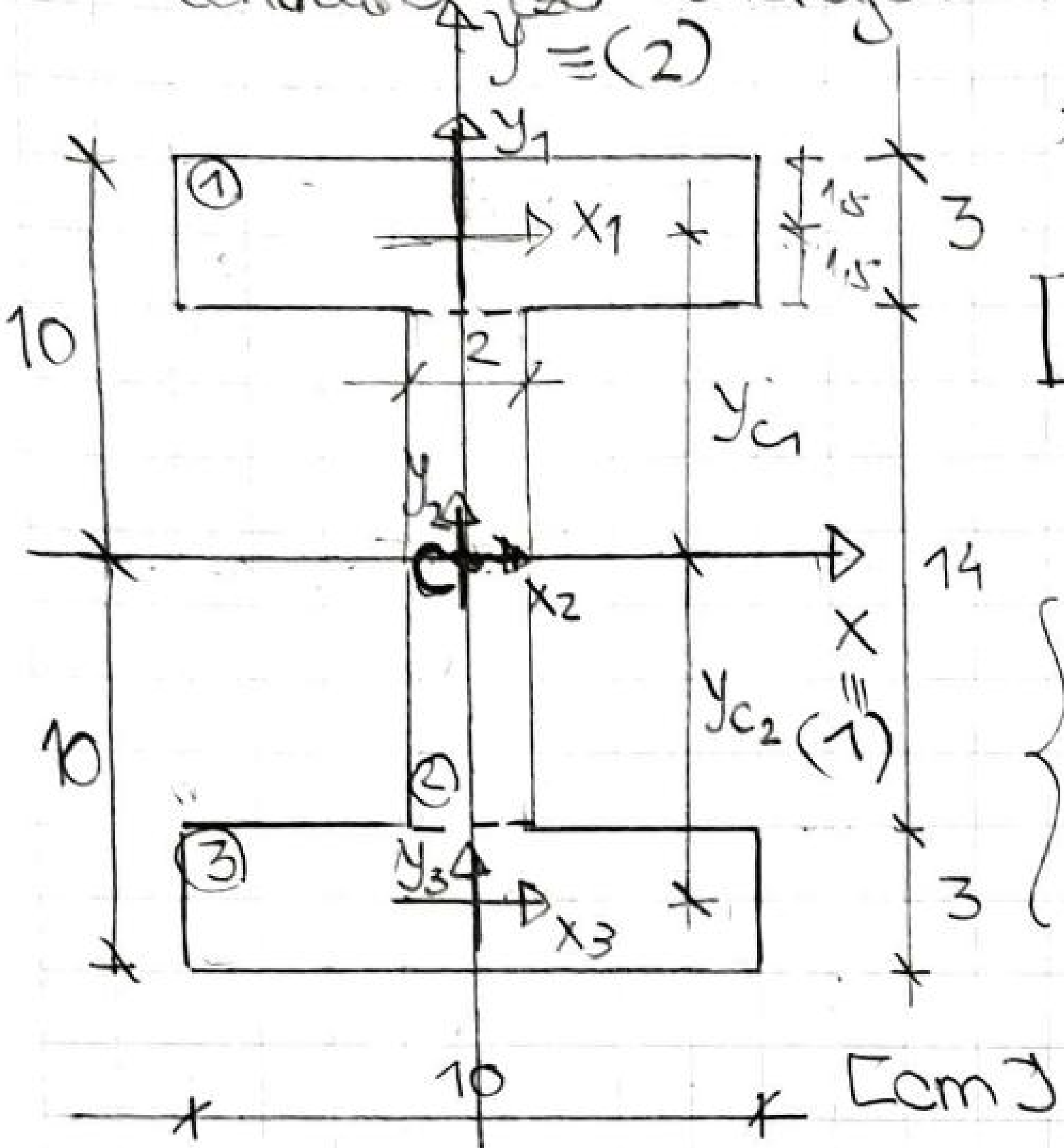
1. Za presjek na slici sračunati glavne centralne momente inercije i nacrtati glavne centralne ose inercije:



x, y – ose simetrije

$$\boxed{I_x = I_x^{(1)} + I_x^{(2)} + I_x^{(3)}}$$ → Štajnerova formula → $b_1 \cdot h_1$

$$I_x^{(1)} = I_{x1}^{(1)} + y_{c1}^2 \cdot A_1 = \frac{h^3 \cdot b}{12} + y_{c1}^2 \cdot A_1$$

$$\boxed{I_x^{(1)}} = \frac{3^3 \cdot 10}{12} + 8{,}5^2 \cdot \underbrace{3 \cdot 10}_{A_1} = \boxed{2190\ cm^4}$$

$$\boxed{I_x^{(2)}} = \frac{14^3 \cdot 2}{12} + 0 \cdot 2 \cdot 14 = \boxed{457{,}3^{\circ}\ cm^4}$$

$$\boxed{I_x^{(3)}} = \frac{3^3 \cdot 10}{12} + (-8{,}5)^2 \cdot 3 \cdot 10 = \boxed{2190\ cm^4}$$

$$\boxed{I_x} = 2190 + 457{,}3^{\circ} + 2190 = \boxed{4837{,}3^{\circ}\ cm^4}$$

$$I_y = I_y^{(1)} + I_y^{(2)} + I_y^{(3)}$$

$$I_y^{(1)} = I_{y1} + x_{c1}^2 \cdot A_1 = \frac{b^3 \cdot h}{12} + x_{c1}^2 \cdot A_1$$

$$\boxed{I_y^{(1)}} = \frac{10^3 \cdot 3}{12} + 0 \cdot 30 = \boxed{250\ cm^4}$$

$$\boxed{I_y^{(2)}} = \frac{2^3 \cdot 14}{12} + 0 \cdot 28 = \boxed{9{,}3^{\circ}\ cm^4}$$

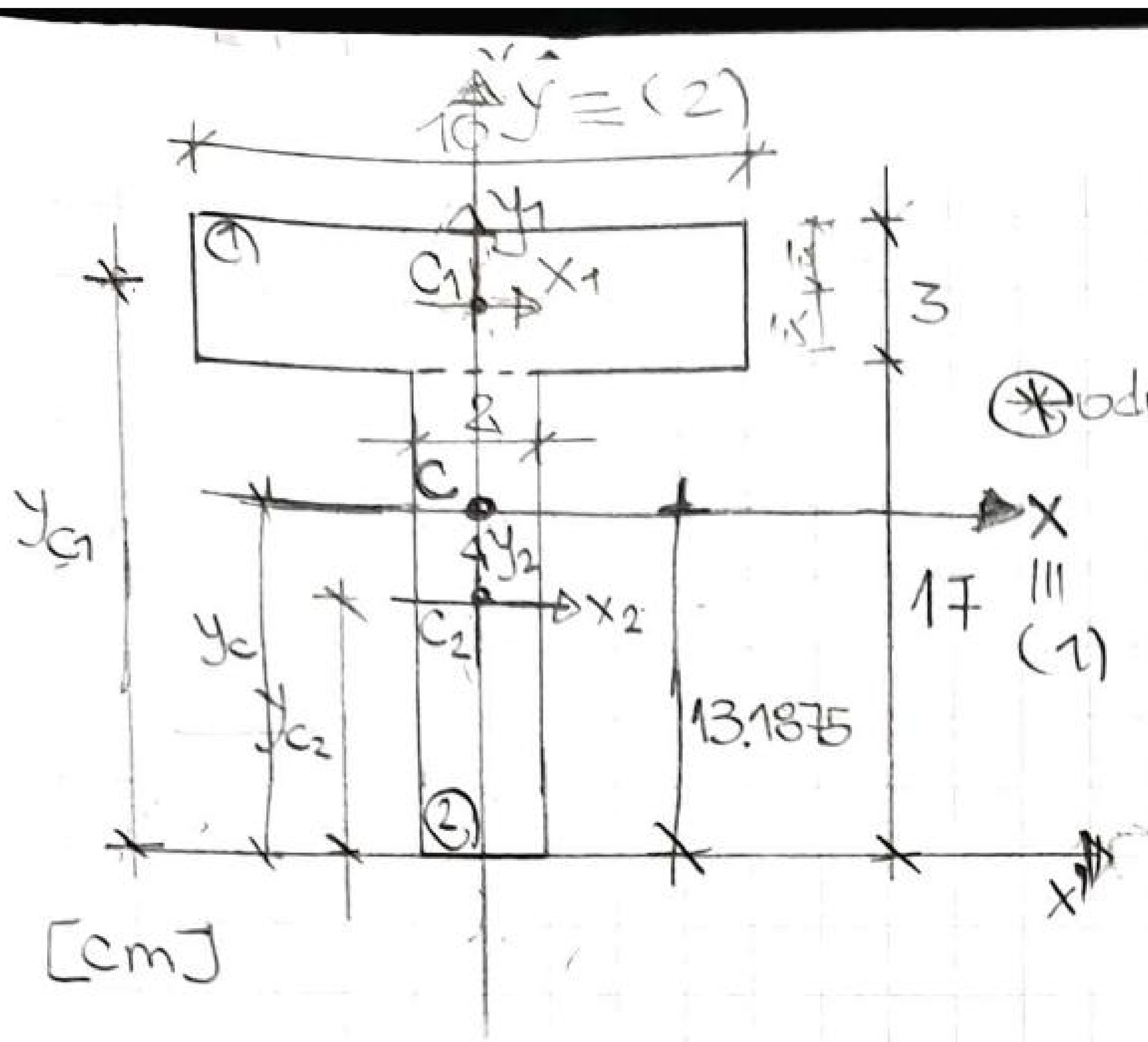
$$\boxed{I_y^{(3)}} = \frac{10^3 \cdot 3}{12} + 0 \cdot 30 = \boxed{250\ cm^4}$$

$$\boxed{I_y} = 250 + 9{,}3^{\circ} + 250 = \boxed{509{,}3^{\circ}\ cm^4}$$

$$\boxed{I_x = I_{1\,(max)} \quad ; \quad I_y = I_{2\,(min)}}$$

$\boxed{I_{xy} = 0}$ (simetričan presjek)

2. Za presjek na slici sračunati glavne centralne momente inercije i nacrtati glavne centralne ose inercije:



$C_1(x_{c1}, y_{c1})$ $C_2(x_{c2}, y_{c2})$

$A_1 = 3 \cdot 10 = 30 cm$

$A_2 = 2 \cdot 17 = 34 cm$

$C_1(0, 18{,}5)$; $C_2(0, 8{,}5)$

* određivanje težišta presjeka

$$x_c = \frac{x_{c1} \cdot A_1 + x_{c2} \cdot A_2}{A_1 + A_2}, \quad y_c = \frac{y_{c1} \cdot A_1 + y_{c2} \cdot A_2}{\underbrace{A_1 + A_2}_{\Sigma A}}$$

$$x_c = \frac{0 \cdot 30 + 0 \cdot 34}{30 + 34} = 0, \quad y_c = \frac{18{,}5 \cdot 30 + 8{,}5 \cdot 34}{64}$$

$$\boxed{y_c} = 13{,}1875 cm$$

$C(0; 13{,}1875)$ – položaj težišta

$$I_x = I_x^{(1)} + I_x^{(2)}$$

$$I_x^{(1)} = I_{x1}^{(1)} + (y_{c1} - y_c)^2 \cdot A_1 = \frac{3^3 \cdot 10}{12} + \underbrace{(\overbrace{18{,}5 - 13{,}1875}^{5.3125})^2}_{662,7} \cdot 30 = 869.18 cm^4$$

$$I_x^{(2)} = \frac{17^3 \cdot 2}{12} + \underbrace{(\overbrace{8{,}5 - 13{,}1875}^{-4.6875})^2}_{744,68} \cdot 34 = 818.8333 + 744{,}68 = 1565{,}90 cm^4$$

$$\boxed{I_x} = 869{,}18 + 1565{,}90 = 2435{,}08 cm^4$$

$$I_y = I_y^{(1)} + I_y^{(2)}$$

$$I_y^{(1)} = I_{y1}^{(1)} + (x_{c1} - x_c)^2 \cdot A_1 = \frac{10^3 \cdot 3}{12} + 0 \cdot A_1 = 250 cm^4$$

y osa – osa simetrije

$$I_y^{(2)} = \frac{2^3 \cdot 17}{12} + 0 \cdot A_2 = 11{,}3\dot{} cm^4$$

$$\boxed{I_y} = 250 + 11{,}3\dot{} = 261{,}3\dot{} cm^4$$

$I_x = I_1$; $I_y = I_2$ → I_1, I_2 – glavne centralne ose inercije

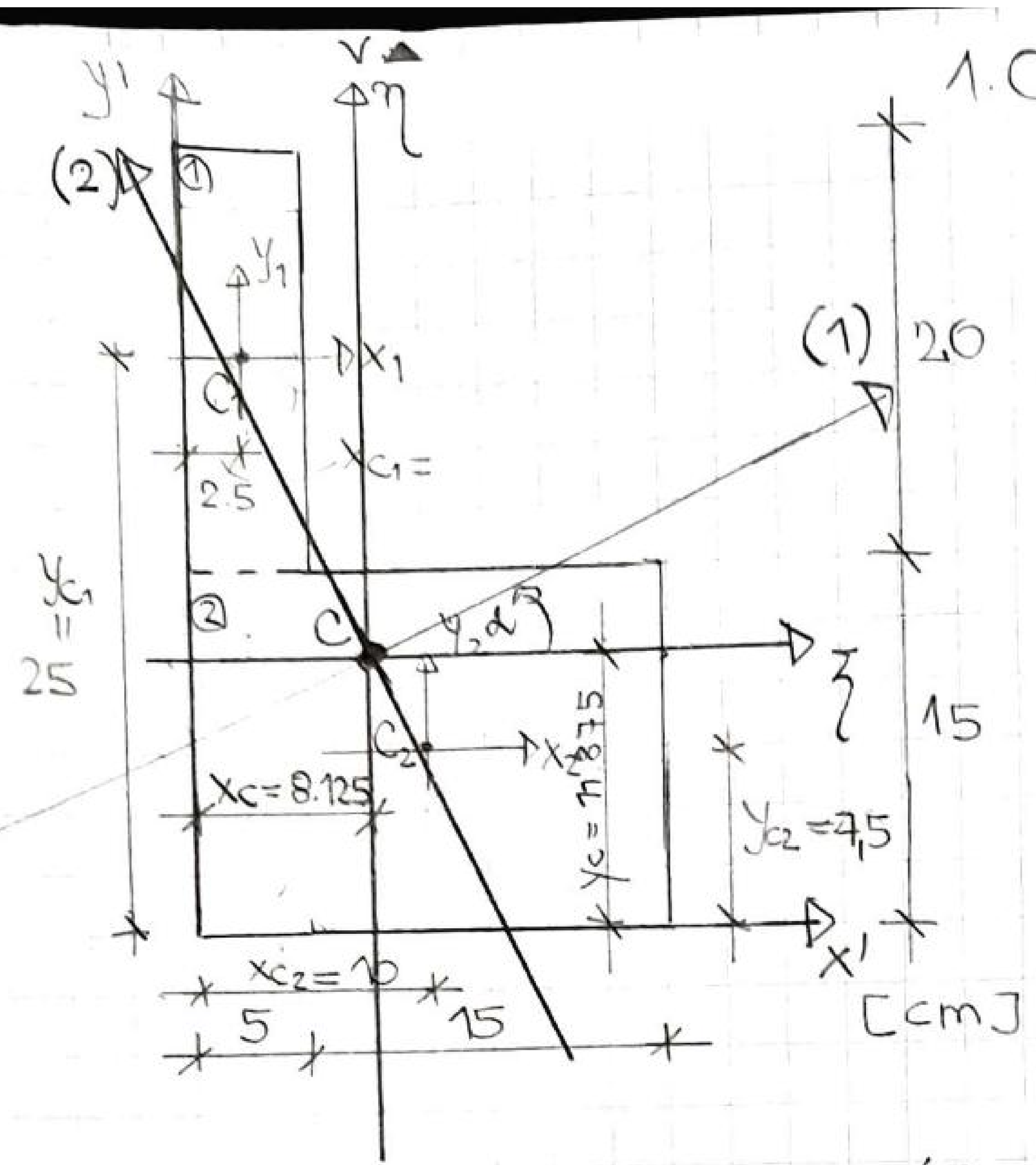
$$\boxed{I_{xy} = 0}$$

3. Za poprečni prosjek na slici odrediti:

a) Glavne centralne momente inercije

b) Glavne centralne ose inercije

c) Morov krug inercije



1. Određivanje težišta

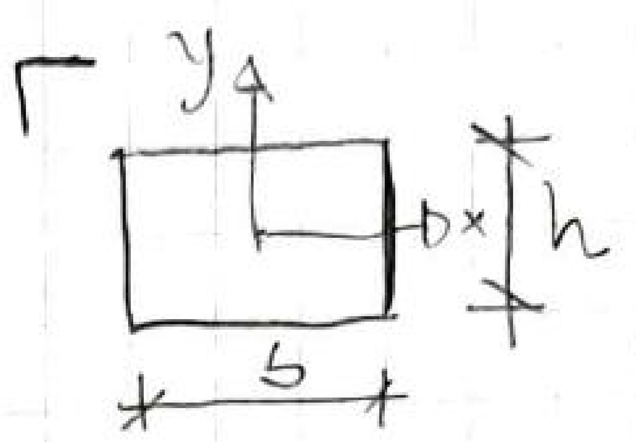
$C_1(2{,}5; 25)$; $C_2(10; 7{,}5)$

$A_1 = 5 \cdot 20 = 100\,cm^2$; $A_2 = 20 \cdot 15 = 300\,cm^2$

$$x_c = \frac{2{,}5 \cdot 100 + 10 \cdot 300}{400} = 8{,}125\,cm$$

$$y_c = \frac{25 \cdot 100 + 7{,}5 \cdot 300}{400} = 11{,}875\,cm$$

$C(8{,}125; 11{,}875)$



$$I_x = \frac{h^3 \cdot b}{12}$$

$$I_y = \frac{b^3 \cdot h}{12}$$

2. * Centralni momenti inercije (I_ξ, I_η)

$$I_\xi = I_\xi^{(1)} + I_\xi^{(2)}$$

$$I_\xi^{(1)} = \frac{20^3 \cdot 5}{12} + (25 - 11{,}875)^2 \cdot 100 = 3333.\dot{3} + 17226.562 = 20\,559{,}8958\ cm^4$$

$$I_\xi^{(2)} = \frac{15^3 \cdot 20}{12} + (7{,}5 - 11.875)^2 \cdot 300 = 5625\,cm^4 + 5742.1875 = 11\,367.1875\ cm^4$$

$$\boxed{I_\xi} = 31\,927.0833\,cm^4$$

$$\boxed{I_\eta = I_\eta^{(1)} + I_\eta^{(2)}}$$

$$I_\eta^{(1)} = \frac{5^3 \cdot 20}{12} + (8{,}125 - 2{,}5)^2 \cdot 100 = 208.\dot{3} + 3164{,}0625 = 3372{,}3958\,cm^4$$

$$I_\eta^{(2)} = \frac{20^3 \cdot 15}{12} + (10 - 8.125)^2 \cdot 300 = 10\,000 + 1054{,}6875 = 11054.6875\ cm^4$$

$$\boxed{I_\eta = 14427.0833\,cm^4}$$

$$\boxed{I_{\xi\eta} = I_{\xi\eta}^{(1)} + I_{\xi\eta}^{(2)}}$$

Znak centrifugalnog momenta *

$$I_{\xi\eta}^{(1)} = I_{x_1y_1} + (y_{c_1} - y_c) \cdot (x_{c_1} - x_c) \cdot A_{c_1} =$$

→ jer y, x ose simetrije

$$= 0 + (25 - 11.875) \cdot (2{,}5 - 8.125) \cdot 100 = -7382{,}8125\,cm^4$$

13.125 ; −5.625

$$I_{\xi\eta}^{(2)} = 0 + (7{,}5 - 11{,}875) \cdot (10 - 8.125) \cdot 300 = -2460{,}937\,cm^4$$

$$\boxed{I_{\xi\eta} = -9843{,}75\,cm^4}$$

* Predznak sopstvenog centrifugalnog momenta proste figure se mora odrediti za svaki pojedinačni slučaj, jer zavisi od položaja u odnosu na lokalni koord. sistem. Predznak se određuje procjenom da li veći dio površine presjeka pripada I i III kvadr. usvojenog lokalnog sistema, gdje je međusobni proizvod koord. pozitivan ($I_{xy} > 0$) ili II i IV kvadrantu sa negativnim predznakom ($I_{xy} < 0$)

*Glavni centralni momenti inercije:

$$I_{1/2} = \frac{1}{2}(I_\xi + I_\eta) \pm \frac{1}{2}\sqrt{(I_\xi - I_\eta)^2 + 4 I_{\xi\eta}^2}$$

$$I_{1/2} = \frac{1}{2}\cdot(31\,927{,}0833 + 14427{,}0833) \pm \frac{1}{2}\sqrt{(31\,927{,}0833 - 14427{,}0833)^2 + 4\cdot(-9843{,}75)^2}$$

$$I_{1/2} = 23\,177{,}0833 \pm 13170.49$$

$$I_1 = 36\,347{,}5773\,cm^4$$

$$I_2 = 10\,006.593\,cm^4$$

*Položaj glavnih centralnih osa inercije:

$$tg\,2\alpha = -\frac{2 I_{\xi\eta}}{I_\xi - I_\eta} = \frac{+2\cdot 9843.75}{31\,927.0833 - 14427.0833}$$

$$tg\,2\alpha = 1.125$$

$$2\alpha = 48.366$$

$$\boxed{\alpha = 24{,}18^0}$$

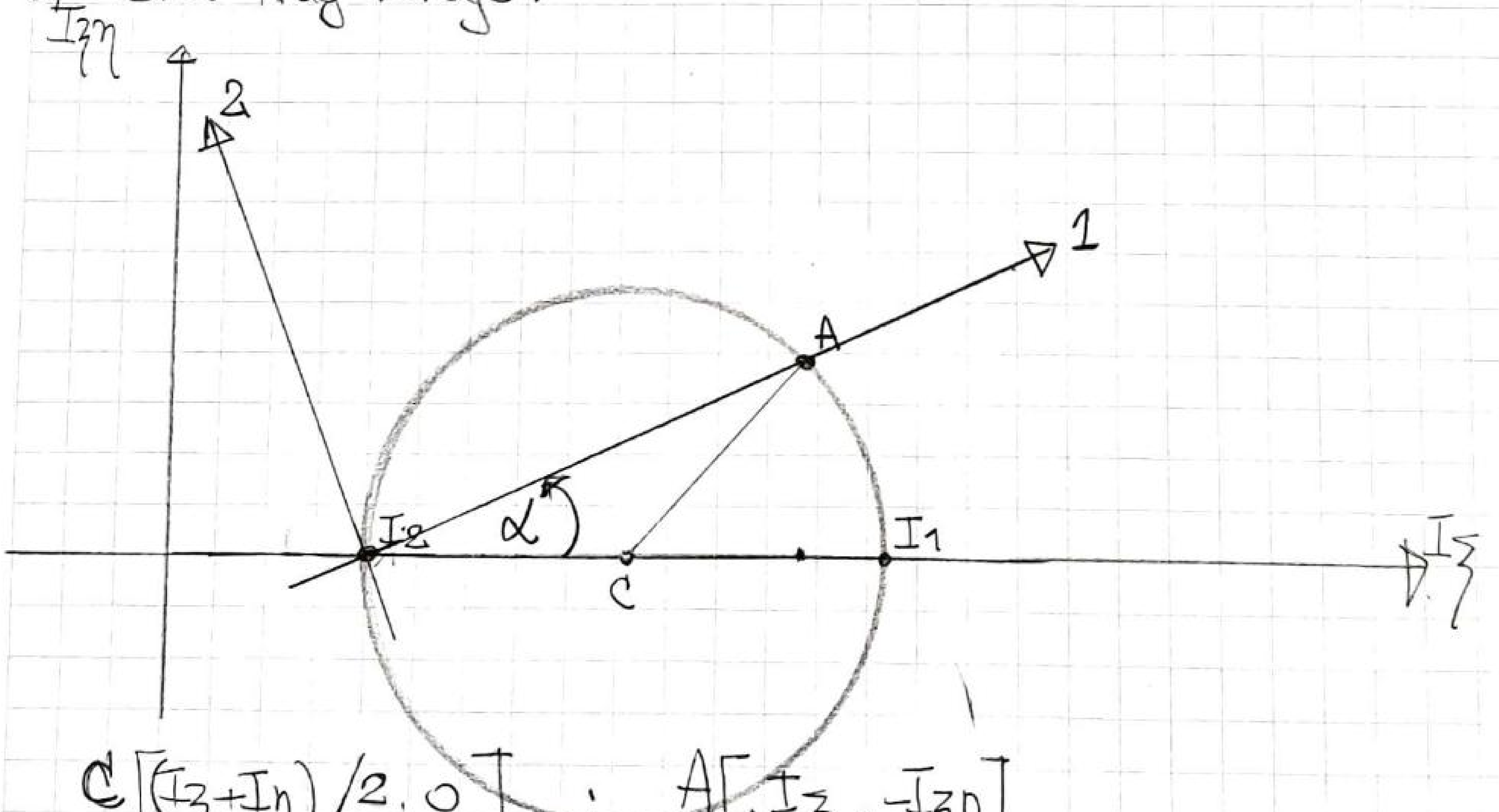
KONTROLA: $I_1 + I_2 = I_\xi + I_\eta$

⌜ I_1, I_2 → uvijek pozitivni

$I_{\xi\eta}$ prostih figura → odrediti znak ⌟

usvaja se: 1cm = 5000kN

c) Morov krug inercije:



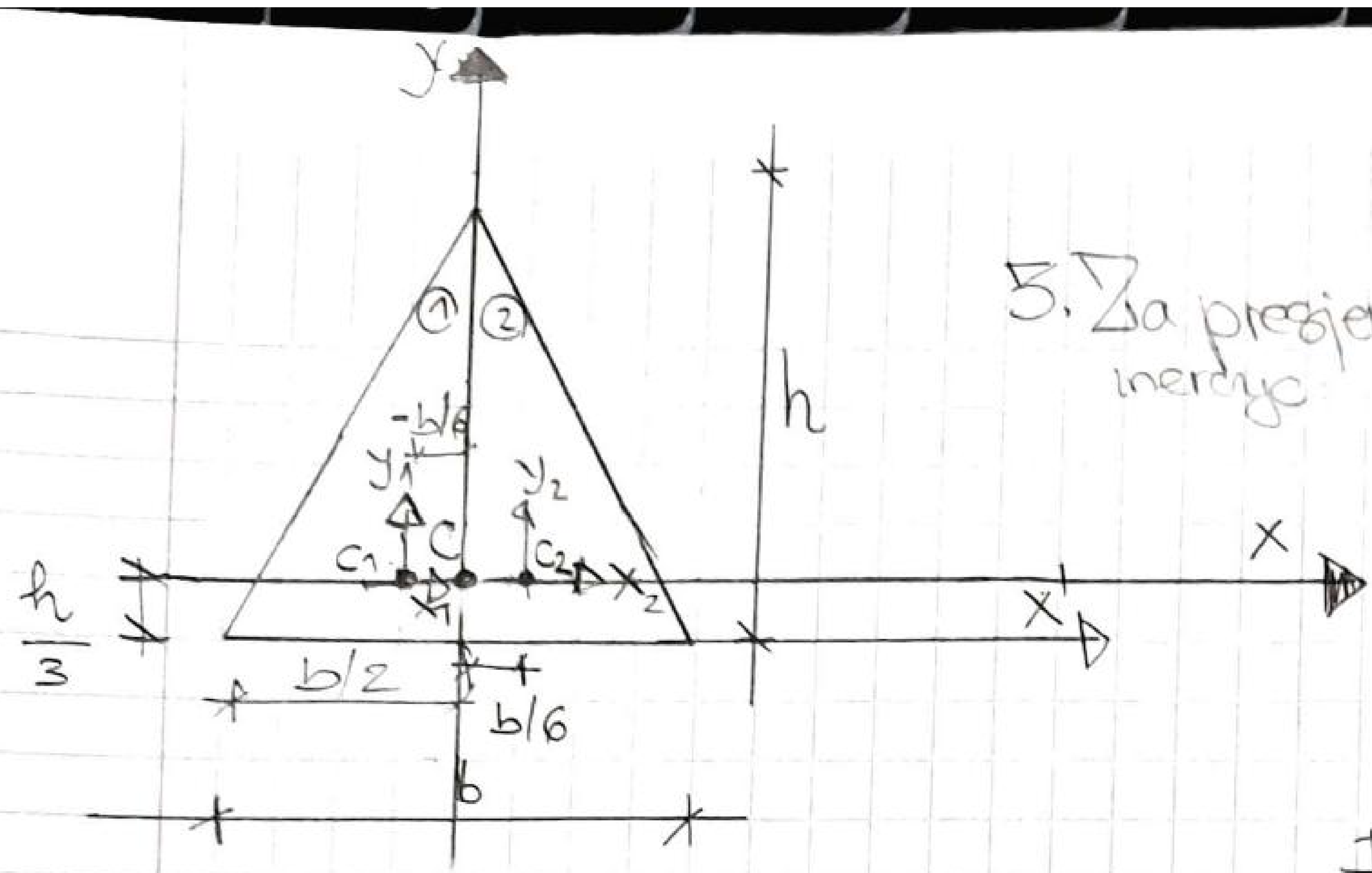
$C[(I_\xi + I_\eta)/2 \,.\, 0]$; $A[I_\xi, -I_{\xi\eta}]$

$C[23177{,}0833 ; 0]$; $A[31927{,}0833 ; 9843{,}75]$

⌜ $\alpha = 0.5\,arctg\left(\frac{-2 I_{\xi\eta}}{I_\xi - I_\eta}\right)$ za $(I_\xi - I_\eta) > 0$

–||– $+90$ za $(I_\xi - I_\eta) < 0$ ⌟

Scanned by TapScanner



5. Za presjek, na slici odrediti centralne momente inercije

$C_1(x_{c1}, y_{c1})$ $\quad A_1 = \frac{1}{2}\frac{b}{2}h = \frac{bh}{4}$

$C_2(x_{c2}, y_{c2})$ $\quad A_2 = \frac{1}{2}\frac{bh}{2} = \frac{bh}{4}$

$C_1\left(-\frac{b}{6}, \frac{h}{3}\right) \quad C_2\left(\frac{b}{6}, \frac{h}{3}\right)$

$$x_c = \frac{-\frac{b}{6}\cdot\frac{b\cdot h}{4} + \frac{b}{6}\cdot\frac{b\cdot h}{4}}{\frac{1}{4}bh + \frac{1}{4}bh}$$

$$\boxed{x_c = 0}$$

$$y_c = \frac{\frac{h}{3}\cdot\frac{bh}{4} + \frac{h}{3}\cdot\frac{bh}{4}}{\frac{bh}{2}} = 0$$

$$y_c = \frac{\frac{2h^2\cdot b}{12}}{\frac{bh}{2}} = \frac{4h}{12} = \frac{h}{3}$$

$$\boxed{y_c = \frac{h}{3}} \quad \boxed{C\left(0, \frac{h}{3}\right)}$$

* Momenti inercije:

$I_x = I_x^{(1)} + I_x^{(2)}$

$I_x^{(1)} = \frac{h^3\cdot b}{72} + 0$

$I_x^{(2)} = \frac{h^3 b}{72} + 0$

$\boxed{I_x} = \frac{h^3 b}{72} + \frac{h^3 b}{72} = \boxed{\frac{h^3 b}{36}}$

$I_y = I_y^{(1)} + I_y^{(2)}$

$I_y^{(1)} = \frac{b^3 h}{8\cdot 36} + \left(-\frac{b}{6} - 0\right)^2 \cdot \frac{bh}{4}$

$I_y^{(1)} = \frac{b^3 h}{288} + \frac{b^3 h}{36\cdot 4}$

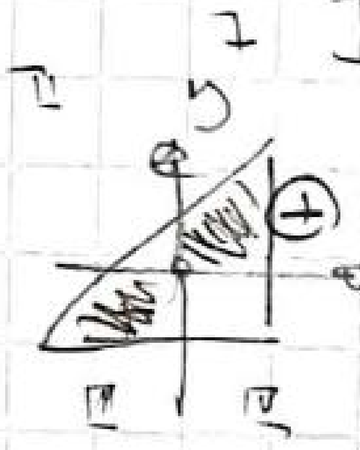
$I_y^{(1)} = \frac{2b^3 h + b^3 h}{288} = \frac{3b^3 h}{288} = \boxed{\frac{b^3 h}{96}}$

$I_y^{(2)} = \frac{b^3\cdot h}{8\cdot 36} + \left(\frac{b}{6} - 0\right)^2 \cdot \frac{b\cdot h}{4}$

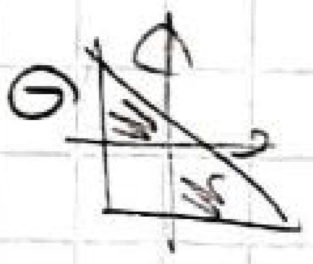
$I_y^{(2)} = \frac{b^3 h}{288} + \frac{b^3 h}{144} = \frac{b^3 h}{96}$

$\underline{I_y} = I_y^{(1)} + I_y^{(2)} = \frac{b^3 h}{96} + \frac{b^3 h}{96} = \boxed{\frac{b^3 h}{48}}$

$I_{xy} = I_{xy}^{(1)} + I_{xy}^{(2)}$



$I_{xy}^{(1)} = +\frac{b^2 h^2}{72} + 0\cdot\left(-\frac{b}{3}\right)\cdot A = \frac{b^2 h^2}{72}$



$I_{xy}^{(2)} = -\frac{b^2 h^2}{72} + 0\cdot\left(\frac{b}{3}\right)\cdot A = -\frac{b^2 h^2}{72}$

$\boxed{I_{xy}} = \frac{b^2 h^2}{72} - \frac{b^2 h^2}{72} = \boxed{0}$

y - osa simetrije